On Some Optimization Problems

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Outline

• **Research background**
  • Optimization

• **Research in Staffordshire University**
  • Mathematical modeling for ore processing in copper mining industry

• **Research in University of Joseph Fourier**
  • Mathematical modeling for itinerary planning
Variational Inequality Problem

For a given nonempty closed convex set \( S \subseteq \mathbb{R}^n \) and a continuous mapping \( F : \mathbb{R}^n \rightarrow \mathbb{R}^n \), the \textit{variational inequality problem}, denoted \( \text{VI}(S, F) \), is to find a vector \( x^* \in S \) such that

\[
\langle F(x^*), x - x^* \rangle \geq 0, \quad \forall x \in S, \tag{1}
\]

where \( \langle \cdot, \cdot \rangle \) denotes the inner product in \( \mathbb{R}^n \).
VIP – Applications

- when $S \equiv \mathbb{R}^n$, (1) $\iff$

$$F(x) = 0.$$  \hspace{1cm} (2)

- when $F(x) \equiv \nabla f(x)$, (1) $\iff$ the optimality condition for

$$\min f(x) \text{ subject to } x \in S.$$  \hspace{1cm} (3)

- Saddle point problems
- Nash equilibrium, Economic equilibrium and Traffic equilibrium problems
- Nonlinear obstacle problems
- Pricing American options
- SPSD matrices optimization
- …
VIP – Solution methods

**Methods for monotone VIPs**

- projection methods, proximal point methods, splitting methods, equation reduction and interior point methods, …

**Methods for not necessarily monotone VIPs**

- KKT based methods
- merit function based methods
  - Lipschitzian branch and bound method
- restricted step Josephy-Newton method
- hybrid evolutionary algorithm
  - (global optimization algorithm)
  - (generalized Nash equilibrium algorithm)
Erdenet Copper Mining Factory

- started its operation in 1978
- Erdenet processes 25 million tons of ore in a year
- another 30 years of resource is left

- produces 126,700 tons of copper and 1,954 tons of molybdenum /2010/
- accounts for 13.5% of GDP
- produces 1% of world copper industry
Copper ore processing

Mining Operation

Multi-stage Crushing and Mixing

Open Pit

1 2 3 4 5

0.50%
0.67%
0.57%
0.35%
0.75%

Copper content 0.535%

Copper extraction /chemical/

E(hour) = 0.535%
Ore crushing and mixing procedure

E(hour) = 0.535%

Open Pit → Big Crusher → Warehouse 1 → Medium Crusher → Screen 1 → Warehouse 2 → Small Crusher → Screen 2 → Warehouse 3 → Moving conveyor → Copper Extraction/Flotation/

Copper content?
Ore crushing and mixing procedure
Ore crushing and mixing procedure

Our goal and approach

• To construct some mathematical model for the ore crushing and mixing process
• To conduct numerical and simulation analysis for the model
• To analyze different ways to conduct the ore crushing and mixing process
46.6% of output ore has copper content in the interval $0.535 \pm 5\%$
Building another warehouse

83.6% of output ore has copper content in the interval 0.535±5%
## Increasing mixture: comparison

<table>
<thead>
<tr>
<th>Procedure</th>
<th>Ore with copper content in the interval 0.535±5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Current</td>
<td>46.6%</td>
</tr>
<tr>
<td>2 Continuous movement of warehouse conveyor</td>
<td>45.7%</td>
</tr>
<tr>
<td>3 Two trucks at a time in Big crusher</td>
<td>61.7%</td>
</tr>
<tr>
<td>4 Using 20 tons trucks</td>
<td>52.2%</td>
</tr>
<tr>
<td>5 Building another warehouse</td>
<td>83.6%</td>
</tr>
</tbody>
</table>
Possibility of predicting the output

Constant and continuous movement of warehouse conveyors
Research in UJF

Itinerary planning problem

A tourist is coming to a city (Paris). The city has attraction points of number $n$, and the tourist has some amount of money (M) to spend on sightseeing under the time period of T. Determine the best route for the tourist concerning the expected satisfaction to be maximal?

Complexity

• If $n=10$, there are 3628800 different routes.
• If the tourist were to choose 7 places out of ten, there are 5040 different routes.
### Mathematical modeling

#### Input

<table>
<thead>
<tr>
<th>Places of Interest</th>
<th>Cost</th>
<th>Time</th>
<th>Expected satisfaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1 (Eiffel Tower)</td>
<td>C1</td>
<td>T1</td>
<td>S1</td>
</tr>
<tr>
<td>A2 (Notre Dame)</td>
<td>C2</td>
<td>T2</td>
<td>S2</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>An (Disneyland)</td>
<td>Cn</td>
<td>Tn</td>
<td>Sn</td>
</tr>
</tbody>
</table>
Mathematical modeling

Transportation cost and time matrices

\[
\begin{pmatrix}
0 & c_{12} & c_{13} & \ldots & c_{1n} \\
c_{12} & 0 & c_{23} & \ldots & c_{2n} \\
\vdots & & & & \vdots \\
c_{1n} & c_{2n} & \ldots & & 0
\end{pmatrix}
\]

\[
\begin{pmatrix}
0 & t_{12} & t_{13} & \ldots & t_{1n} \\
t_{12} & 0 & t_{23} & \ldots & t_{2n} \\
\vdots & & & & \vdots \\
t_{1n} & t_{2n} & \ldots & & 0
\end{pmatrix}
\]

Optimization model

Sum(satisfaction) ---- > maximize

Sum(sightseeing time) + Sum(trans. time) <= Time
Sum(sightseeing cost) + Sum(trans. cost) <= Cost
Mathematical modeling

Complexity

• Scheduling
• Constraint satisfaction problem
• Assignment problem

Approaches for the solution

• Deterministic (problem specific algorithm)
• Genetic algorithm
• Simulation based optimization
Thank you for your attention!

Questions & Comments?